Cost minimization properties exercise

Only one of these 3 cost functions is consistent with the firm's cost minimization problem. Select which one and explain why the others fail.

1. $c(w, r, q) = 2\frac{w}{r}q^2$
2. $c(w, r, q) = 2(w + r)^2 q^2$
3. $c(w, r, q) = 2(w + r)q^2$

Solution

The first case is discarded because it is homogeneous of degree 0 in input prices. While the cost function should be homogeneous of degree 1 in input costs:

$$c(hw, hr, q) = 2\frac{hw}{hr}q^2 = h^0 c(w, r, q)$$

On the other hand, it is also discarded because it is not increasing in r:

$$\frac{\partial c}{\partial r} = -2\frac{w}{r^2}q^2 < 0$$

This means that an increase in the price of capital leads to a lower cost.
The second case is also discarded since, if we obtain the degree of homogeneity:

$$c(hw, hr, q) = 2(hw + hr)^2 q^2 = h^2 c(w, r, q)$$

We have a cost function homogeneous of degree 2 in the price of inputs. We can also see that it is not a concave function in input prices because the sum of the inputs is squared, making the function convex in input prices.

The correct choice is the third one, as it meets the requirements of being homogeneous of degree 1 and the other properties of minimum cost functions.